

# Virtual Photon Structure Functions to NNLO in QCD

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## Abstract

The unpolarized virtual photon structure functions  $F_2^\gamma(x, Q^2, P^2)$  and  $F_L^\gamma(x, Q^2, P^2)$  are investigated in perturbative QCD for the kinematical region  $\Lambda^2 \ll P^2 \ll Q^2$ , where  $-Q^2(-P^2)$  is the mass squared of the probe (target) photon and  $\Lambda$  is the QCD scale parameter. In the framework of operator product expansion supplemented by the renormalization group method, the definite predictions are derived for the moments of  $F_2^\gamma(x, Q^2, P^2)$  up to the next-to-next-to-leading order (the order  $\alpha_s^2$ ) and for the moments of  $F_L^\gamma(x, Q^2, P^2)$  up to the next-to-leading order (the order  $\alpha_s$ ).

## 1 Introduction

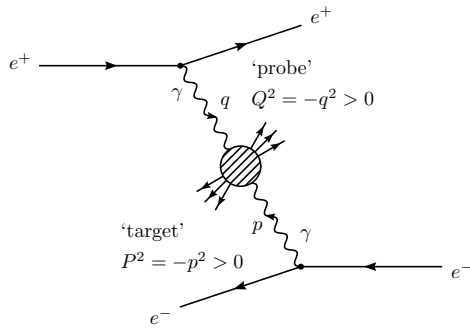


Figure 1: Deep-inelastic scattering on a virtual photon in the  $e^+e^-$  collider experiments

In  $e^+e^-$  collision experiments, the cross

\*Presented by Ken Sasaki.

section for the two-photon processes  $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ , shown in Fig.1, dominates over other processes such as the annihilation process  $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$  at high energies. In particular, the two-photon processes in the double-tag events, where one of the virtual photon is very far off shell (large  $Q^2 \equiv -q^2$ ) while the other is close to the mass shell (small  $P^2 \equiv -p^2$ ), can be viewed as deep-inelastic electron-photon scattering and provide us the information on the structure of the photon. The unpolarized (spin-averaged) photon structure functions  $F_2^\gamma(x, Q^2)$  and  $F_L^\gamma(x, Q^2)$  of the real photon ( $P^2 \approx 0$ ) have been studied in perturbative QCD (pQCD). A pioneering work was done by Witten [1] in which he derived the leading order (LO) QCD contributions to  $F_2^\gamma$  and  $F_L^\gamma$ . A few years later, the next-to-leading order (NLO) corrections to  $F_2^\gamma$  were calculated [2].

A unique and interesting feature of the photon structure functions is that, in contrast with the nucleon case, the target mass squared  $-P^2$  is not fixed but can take various values and that the structure functions show different behaviors depending on the values of  $P^2$ . The photon has two characters: The photon couples directly to quarks (pointlike nature) and sometimes it behaves as vector bosons (hadronic nature). Thus the structure function  $F_2^\gamma(x, Q^2)$  of real photon ( $P^2=0$ ) is composed of a pointlike piece and a hadronic piece. The pointlike part, can be calculated, in principle, in a perturbative method. On the other hand, the hadronic

part, can only be computed by some non-perturbative method like lattice QCD, or estimated by vector meson dominance model. The LO contribution to  $F_2^\gamma(x, Q^2)$ , which behaves as  $1/\alpha_s(Q^2) \sim \ln(Q^2/\Lambda^2)$ , comes from the pointlike part, while the NLO corrections result from both the pointlike and hadronic parts. In terms of the moments, the hadronic energy-momentum tensor operator comes into play at  $n=2$ . Because of the conservation of this operator, the hadronic part gives a finite but perturbatively incalculable contribution at  $n=2$ . The fact that definite information on the NLO second moment is missing prevents us from fully predicting the shape and magnitude of the structure function of  $F_2^\gamma(x, Q^2)$  up to the order  $\mathcal{O}(\alpha)$ .

The situation changes significantly when we analyze the structure function of a virtual photon with  $P^2$  much larger than the QCD parameter  $\Lambda^2$  [3]. More specifically, we consider the following kinematical region,

$$\Lambda^2 \ll P^2 \ll Q^2. \quad (1)$$

In this region, the hadronic component of the photon can also be dealt with *perturbatively* and thus a definite prediction of the whole structure function, its shape and magnitude, may become possible. In fact, the virtual photon structure function  $F_2^\gamma(x, Q^2, P^2)$  in the kinematical region (1) was calculated in the LO (the order  $\alpha/\alpha_s$ ) [4] and in the NLO (the order  $\alpha$ ) [3, 5], and the longitudinal structure function  $F_L^\gamma(x, Q^2, P^2)$  in the LO (the order  $\alpha$ ) [3] without any unknown parameters.

In this talk I report our investigation [6] of the virtual photon structure functions  $F_2^\gamma(x, Q^2, P^2)$  up to the next-to-next-to-leading order (NNLO) (the order  $\alpha\alpha_s$ ) and  $F_L^\gamma(x, Q^2, P^2)$  up to the NLO (the order  $\alpha\alpha_s$ ) in the kinematical region (1).

## 2 $F_2^\gamma(x, Q^2, P^2)$ up to NNLO

We have used the framework of the operator product expansion (OPE) supplemented by

the renormalization group (RG) method. We find that the  $n$ -th moment of  $F_2^\gamma(x, Q^2, P^2)$  for the kinematical region (1) is expressed, up to NNLO, as

$$\begin{aligned} & \int_0^1 dx x^{n-2} F_2^\gamma(x, Q^2, P^2) / \left( \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \right) \\ &= \left\{ \frac{4\pi}{\alpha_s(Q^2)} \sum_i \mathcal{L}_i^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n+1} \right] \right. \\ & \quad + \sum_i \mathcal{A}_i^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n} \right] \\ & \quad + \sum_i \mathcal{B}_i^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n+1} \right] + \mathcal{C}^n \\ & \quad + \frac{\alpha_s(Q^2)}{4\pi} \left( \sum_i \mathcal{D}_i^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n-1} \right] \right. \\ & \quad + \sum_i \mathcal{E}_i^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n} \right] \\ & \quad + \sum_i \mathcal{F}_i^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n+1} \right] + \mathcal{G}^n \left. \right\} + \mathcal{O}(\alpha_s^2), \quad \text{with } i = +, -, NS, \quad (2) \end{aligned}$$

where  $d_i^n = \lambda_i^n/2\beta_0$  and  $\lambda_i^n$  ( $i = +, -, NS$ ) denotes the eigenvalues of 1-loop anomalous dimension matrices. The terms with  $\mathcal{L}_i^n$  are the LO ( $\alpha/\alpha_s$ ) contributions [1]. The NLO ( $\alpha$ ) corrections are the terms with  $\mathcal{A}_i^n$ ,  $\mathcal{B}_i^n$  and  $\mathcal{C}^n$  [2, 3]. The coefficients  $\mathcal{D}_i^n$ ,  $\mathcal{E}_i^n$ ,  $\mathcal{F}_i^n$  and  $\mathcal{G}^n$  give the NNLO ( $\alpha\alpha_s$ ) corrections and they are new. The explicit expressions of  $\mathcal{D}_i^n$ ,  $\mathcal{E}_i^n$ ,  $\mathcal{F}_i^n$  and  $\mathcal{G}^n$  are given in Eqs.(2.34)-(2.37) of Ref.[6] and they are written in terms of the 1-, 2- and 3-loop anomalous dimensions, the 1- and 2-loop coefficient functions and the 1- and 2-loop photon matrix elements of hadronic operators.

For the 3-loop anomalous dimensions, we could use the recently calculated results of the three-loop anomalous dimensions for the quark and gluon operators [7, 8] and of the three-loop photon-quark and photon-gluon splitting functions [9]. The 2-loop photon

matrix elements of hadronic operators were derived from the results of the two-loop operator matrix elements calculated up to the finite terms [10] by changing color-group factors.

We examine the sum rule of  $F_2^\gamma(x, Q^2, P^2)$ , i.e., the second moment, numerically. The NNLO corrections are found to be 7% ~ 10% of the sum of the LO and NLO contributions, when  $P^2 = 1\text{GeV}^2$  and  $Q^2 = 30 \sim 100\text{GeV}^2$  or  $P^2 = 3\text{GeV}^2$  and  $Q^2 = 100\text{GeV}^2$ , and  $n_f$  is three or four.

Next we perform the inverse Mellin transform of (2) to obtain  $F_2^\gamma$  as a function of  $x$ . The  $n$ -th moment is denoted as

$$M_2^\gamma(n, Q^2, P^2) = \int_0^1 dx x^{n-1} \frac{F_2^\gamma(x, Q^2, P^2)}{x}. \quad (3)$$

Then by inverting the moments (3) we get

$$\frac{F_2^\gamma(x, Q^2, P^2)}{x} = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} dn x^{-n} M_2^\gamma(n, Q^2, P^2), \quad (4)$$

where the integration contour runs to the right of all singularities of  $M_2^\gamma(n, Q^2, P^2)$  in the complex  $n$ -plane.

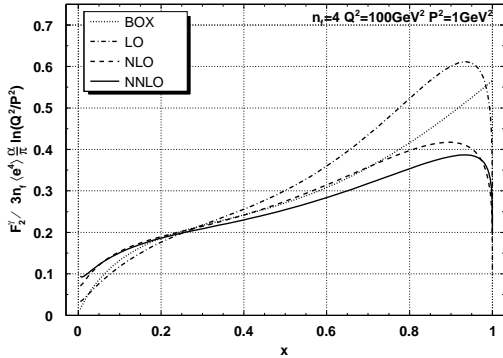


Figure 2: Virtual photon structure function  $F_2^\gamma(x, Q^2, P^2)$  for  $Q^2 = 100 \text{ GeV}^2$  and  $P^2 = 1 \text{ GeV}^2$  with  $n_f = 4$  and  $\Lambda = 0.2 \text{ GeV}$ .

The LO, NLO and NNLO QCD results, as well as the box contribution, for the case

of  $Q^2 = 100 \text{ GeV}^2$  and  $P^2 = 1 \text{ GeV}^2$  with  $n_f = 4$ , are shown in Fig.2. We observe that there exist notable NNLO QCD corrections at larger  $x$ . The corrections are negative and the NNLO curve comes below the NLO one in the region  $0.3 \lesssim x < 1$ . At the lower  $x$  region,  $0.05 \lesssim x \lesssim 0.3$ , the NNLO corrections to the NLO results are found to be negligibly small.

### 3 $F_L^\gamma(x, Q^2, P^2)$ up to NLO

The formula for the  $n$ -th moment of the longitudinal structure function  $F_L^\gamma(x, Q^2, P^2)$  can be obtained from (2) only by replacing the hadronic and photonic coefficient functions  $C_{2,n}(1, \alpha_s)$  and  $C_{2,n}^\gamma(1, \alpha_s, \alpha)$  with the longitudinal counterparts  $C_{L,n}(1, \alpha_s)$  and  $C_{L,n}^\gamma(1, \alpha_s, \alpha)$ , respectively. Since there is no contribution of the tree diagrams to the hadronic longitudinal coefficient functions (and thus we get  $C_{L,n}^{(0)} = 0$  in the expansion of  $C_{L,n}(1, \alpha_s)$ ), the moments of  $F_L^\gamma$  starts at the order  $\alpha$ . The  $n$ -th moment is given as follows:

$$\begin{aligned} & \int_0^1 dx x^{n-2} F_L^\gamma(x, Q^2, P^2) / \left( \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \right) \\ &= \left\{ \sum_i \mathcal{B}_{(L),i}^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n + 1} \right] + \mathcal{C}_{(L)}^n \right. \\ &+ \frac{\alpha_s(Q^2)}{4\pi} \left( \sum_i \mathcal{E}_{(L),i}^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n} \right] \right. \\ &+ \left. \sum_i \mathcal{F}_{(L),i}^n \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{d_i^n + 1} \right] + \mathcal{G}_{(L)}^n \right) \\ &\left. + \mathcal{O}(\alpha_s^2) \right\}, \quad \text{with } i = +, -, NS, \quad (5) \end{aligned}$$

The coefficients  $\mathcal{B}_{(L),i}^n$  and  $\mathcal{C}_{(L)}^n$  represent the LO terms [1, 2, 3], while the terms with  $\mathcal{E}_{(L),i}^n$ ,  $\mathcal{F}_{(L),i}^n$  and  $\mathcal{G}_{(L)}^n$  are the NLO ( $\alpha\alpha_s$ ) corrections and they are new. The explicit expressions of  $\mathcal{E}_{(L),i}^n$ ,  $\mathcal{F}_{(L),i}^n$  and  $\mathcal{G}_{(L)}^n$  are given in Eqs.(6.6)-(6.8) of Ref.[6].

Inverting the moments (5), we plot in Fig.3 the longitudinal virtual photon structure function  $F_L^\gamma(x, Q^2, P^2)$  predicted by pQCD for the case of  $n_f = 4$ ,  $Q^2 = 100 \text{ GeV}^2$  and  $P^2 = 1 \text{ GeV}^2$ . We show three curves; the LO and NLO QCD results and the Box (tree) diagram contribution. We see that the NLO QCD corrections are negative and the NLO curve comes below the LO one in the region  $0.2 \lesssim x < 1$ .

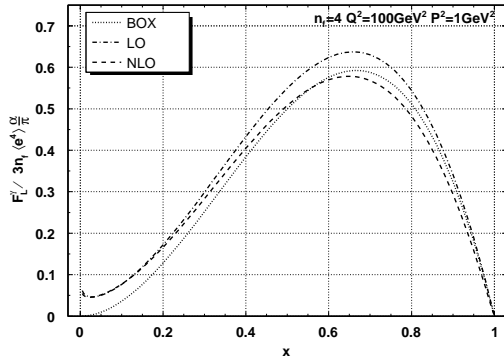


Figure 3: Longitudinal photon structure function  $F_L^\gamma(x, Q^2, P^2)$  for  $Q^2 = 100 \text{ GeV}^2$  and  $P^2 = 1 \text{ GeV}^2$  with  $n_f = 4$  and  $\Lambda = 0.2 \text{ GeV}$ .

## 4 Conclusions

We have investigated the unpolarized virtual photon structure functions  $F_2^\gamma(x, Q^2, P^2)$  and  $F_L^\gamma(x, Q^2, P^2)$  for the kinematical region  $\Lambda^2 \ll P^2 \ll Q^2$  in QCD. In the framework of the OPE supplemented by the RG method, we gave the definite predictions for the moments of  $F_2^\gamma(x, Q^2, P^2)$  up to the NNLO (the order  $\alpha\alpha_s$ ) and for the moments of  $F_L^\gamma(x, Q^2, P^2)$  up to the NLO (the order  $\alpha\alpha_s$ ). In the course of our evaluation, we utilized the recently calculated results of the three-loop anomalous dimensions for the quark and gluon operators. Also we derived the photon matrix elements of hadronic operators up to the two-loop level.

The inverse Mellin transform of the moments was performed to express the structure functions  $F_2^\gamma(x, Q^2, P^2)$  and  $F_L^\gamma(x, Q^2, P^2)$  as functions of  $x$ . We found that there exist sizable NNLO contributions for  $F_2^\gamma$  at larger  $x$ . The corrections are negative and the NNLO curve comes below the NLO one in the region  $0.3 \lesssim x < 1$ . At lower  $x$  region,  $0.05 \lesssim x \lesssim 0.3$ , the NNLO corrections to the NLO results are found to be negligibly small. Concerning  $F_L^\gamma$ , the NLO corrections reduce the magnitude in the region  $0.2 \lesssim x < 1$ .

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